Renormalizing Coupled Scalars with a Momentum Dependent Mixing Angle in the MSSM [*]

Marco Aurelio Díaz

Department of Physics and Astronomy

Vanderbilt University, Nashville, TN 37235

Abstract

The renormalization of a system of coupled scalars fields is analyzed. By introducing a momentum dependent mixing angle we diagonalize the inverse propagator matrix at any momentum p^2. The zeros of the inverse propagator matrix, *i.e.*, the physical masses, are then calculated keeping the full momentum dependence of the self energies. The relation between this method and others previously published is studied. This idea is applied to the one-loop renormalization of the CP-even neutral Higgs sector of the Minimal Supersymmetric Model, considering top and bottom quarks and squarks in the loops.

Several years ago, Capdequi-Peyranère and Talon[1] studied the wave function renormalization of coupled systems of scalars, fermions, and vectors. Their approach includes conventional mass counterterms and wave function renormalization plus a non-conventional field "rotation" that allow them to impose no mixing between the states at different scales, those scales being the masses of the different states. Here we generalize this idea and, at the same time, explain the nature of this field "rotation".

Similarly to ref. [1], consider the bare lagrangian corresponding to a system of two scalars:

$$\mathcal{L}_b = \tfrac{1}{2}\chi_{1b}(p^2 - m_{1b}^2)\chi_{1b} + \tfrac{1}{2}\chi_{2b}(p^2 - m_{2b}^2)\chi_{2b} - \chi_{1b}m_{12b}^2\chi_{2b} \tag{1}$$

If we denote by $-iA_{ij}^\chi(p^2)$, $i,j = 1,2$, the sum of the one-loop Feynman graphs contributing to the two point functions and, after shifting the bare masses by $m_{ib}^2 \to m_i^2 - \delta m_i^2$, $i = 1,2,12$, and the fields by $\chi_{ib} \to (1 + \tfrac{1}{2}\delta Z_i)\chi_i$, the effective lagrangian is

$$\mathcal{L}_{eff} = \tfrac{1}{2}(\chi_1, \chi_2)\boldsymbol{\Sigma^\chi}\begin{pmatrix}\chi_1 \\ \chi_2\end{pmatrix} \tag{2}$$

with

$$\begin{aligned}
\Sigma_{11}^\chi(p^2) &= p^2 - m_1^2 + (p^2 - m_1^2)\delta Z_1 + \delta m_1^2 - A_{11}^\chi(p^2) \\
\Sigma_{22}^\chi(p^2) &= p^2 - m_2^2 + (p^2 - m_2^2)\delta Z_2 + \delta m_2^2 - A_{22}^\chi(p^2) \\
\Sigma_{12}^\chi(p^2) &= -m_{12}^2 - \tfrac{1}{2}m_{12}^2(\delta Z_1 + \delta Z_2) + \delta m_{12}^2 - A_{12}^\chi(p^2)
\end{aligned} \tag{3}$$

Although it is not a necessary assumption, in order to compare more easily with ref. [1], we assume for the moment that the two scalars are diagonal at tree level, i.e., $m_{12}^2 = 0$. In this case, if we want the pole of the propagators to be the physical masses with a residue equal to unity, the two mass counterterms and the two wave function renormalization are fixed through the relations

$$\begin{aligned}
\delta m_1^2 &= A_{11}^\chi(m_1^2), & \delta m_2^2 &= A_{22}^\chi(m_2^2) \\
\delta Z_1 &= A_{11}'^\chi(m_1^2), & \delta Z_2 &= A_{22}'^\chi(m_2^2)
\end{aligned} \tag{4}$$

where the prime denote the derivative with respect to the argument. We may want

to fix the δm_{12}^2 counterterm by imposing no mixing between χ_1 and χ_2 at a given scale, for example at $p^2 = m_1^2$. In this case, the off diagonal element of the inverse propagator matrix is

$$\Sigma_{12}^\chi(p^2) = \delta m_{12}^2 - A_{12}^\chi(p^2) = A_{12}^\chi(m_1^2) - A_{12}^\chi(p^2) \equiv -\tilde{A}_{12}^\chi(p^2) \tag{5}$$

At this point all the counterterms are fixed, and since $\tilde{A}_{12}^\chi(p^2)$ is zero only at $p^2 = m_1^2$, the two fields are not decoupled at a different scale. This is the motivation for the authors in ref. [1] to define the unconventional wave function renormalization $\chi_{1b} \to (1-\alpha_1)\chi_1 - \beta_1\chi_2$ and $\chi_{2b} \to (1-\alpha_2)\chi_2 - \beta_2\chi_1$ instead of $\chi_{1b} \to (1+\frac{1}{2}\delta Z_i)\chi_i$ we use here. Setting to one the residue of the pole of each propagator they find

$$\alpha_i = -\tfrac{1}{2}\delta Z_i = -\tfrac{1}{2}A_{ii}^{\prime\chi}(m_1^2), \qquad i = 1, 2 \tag{6}$$

and imposing no mixing between the two fields at $p^2 = m_1^2$ and *also* at $p^2 = m_2^2$ they get

$$\beta_1 = \frac{A_{12}^\chi(m_2^2)}{m_1^2 - m_2^2}, \qquad \beta_2 = \frac{A_{12}^\chi(m_1^2)}{m_2^2 - m_1^2} \tag{7}$$

In the language we are using here, the later is equivalent to perform a "rotation" (we already know that it is not a field rotation, it is just a wave function renormalization that mixes the two fields) to the inverse propagator matrix in the following way

$$\mathbf{\Sigma^\chi} \longrightarrow \begin{bmatrix} 1 & \beta_2 \\ \beta_1 & 1 \end{bmatrix} \mathbf{\Sigma^\chi} \begin{bmatrix} 1 & \beta_1 \\ \beta_2 & 1 \end{bmatrix} \tag{8}$$

Here we propose a modification of this procedure. If we define a momentum dependent mixing angle we will be able to diagonalize the inverse propagator matrix at *any* momentum[2]. Considering the already finite inverse propagator matrix

3

elements in eq. (3), we define a momentum dependent mixing angle $\alpha(p^2)$ by

$$\tan[2\alpha(p^2)] = \frac{2\Sigma_{12}^{\chi}(p^2)}{\Sigma_{11}^{\chi}(p^2) - \Sigma_{22}^{\chi}(p^2)} \tag{9}$$

The matrix $\boldsymbol{\Sigma}^{\chi}(p^2)$ is diagonalized at any momentum p^2 by a rotation defined by the angle $\alpha(p^2)$

$$\boldsymbol{\Sigma}^{\chi} \longrightarrow \begin{bmatrix} c_{\alpha} & s_{\alpha} \\ -s_{\alpha} & c_{\alpha} \end{bmatrix} \boldsymbol{\Sigma}^{\chi} \begin{bmatrix} c_{\alpha} & -s_{\alpha} \\ s_{\alpha} & c_{\alpha} \end{bmatrix} \tag{10}$$

where s_{α} and c_{α} are sine and cosine of the momentum dependent mixing angle $\alpha(p^2)$. Considering eqs. (3), (5), and (9) we find in first approximation

$$s_{\alpha}(p^2) \approx \frac{\tilde{A}_{12}^{\chi}(p^2)}{m_2^2 - m_1^2}, \qquad c_{\alpha} \approx 1 \tag{11}$$

making evident the relation with the previous method.

There is a fundamental difference between our approach and the one in ref. [1]: we are rotating an already finite inverse propagator matrix with a rotation matrix defined by a finite momentum dependent mixing angle, on the contrary, in ref. [1] the "rotation" is in fact a wave function renormalization that mixes the two fields and the "rotation" matrix elements β_1 and β_2 are infinite. A momentum dependent field rotation given by eqs. (9) and (10) is the only way to diagonalize the inverse propagator matrix at any scale. On the other hand, the only momentum independent way to diagonalize this matrix at two scales ($p^2 = m_1^2$ and $p^2 = m_2^2$) is with the field "rotation" in eqs. (7) and (8). By contrast, the conventional renormalization of this matrix will allow us to diagonalize it at only one scale. According to our example in eq. (5), that scale is $p^2 = m_1^2$, and any further momentum independent rotation of the fields will diagonalize the inverse propagator matrix at a different scale, for example at $p^2 = m_2^2$, by using $\alpha(p^2 = m_2^2)$ in eq. (11), but spoiling the previous diagonalization (at $p^2 = m_1^2$).

Using a momentum dependent mixing angle in this way is an alternative to define a counterterm for this angle. In fact, the renormalization procedure is carried out in the unrotated basis and no mixing angle is defined at that level. Similarly, instead of renormalizing couplings of the rotated fields to other particles, we renormalize couplings of the unrotated fields to those particles and after that we rotate by an angle $\alpha(p^2)$, where p^2 is the typical scale of the process, for example, $p^2 = m_i^2$ if the rotated field χ_i is on-shell. Usually, working out the radiative corrections in the unrotated basis implies one extra advantage, and that is the simplicity of the Feynman rules. In the following we will illustrate these ideas by renormalizing the CP-even neutral Higgs masses of the Minimal Supersymmetric Model (MSSM).

The radiative corrections to the Higgs masses in the MSSM have been studied by many authors in the last three years, using different techniques and focusing in different particles and processes. It was established the theoretical convenience of parametrizing the Higgs sector through the CP-odd Higgs mass (m_A) and the ratio of the vacuum expectation values of the two Higgs doublets $(\tan\beta = v_2/v_1)$. The radiative corrections to the charged Higgs mass were found to be small[3], growing as m_t^2, unless there is an appreciable mixing in the squark mass matrix: in that case a term proportional to m_t^4 is non-negligible. The corrections to the CP-even Higgs masses are large and grow as m_t^4, and have profound consequences in the phenomenology of the Higgs sector[4].

We renormalize the CP-even Higgs sector of the MSSM working in an on-shell type of scheme[5], where the physical masses of the gauge bosons m_Z and m_W, and of the CP-odd Higgs m_A correspond to the pole of the propagators. The parameter $\tan\beta$ is defined through the renormalization of the CP-odd Higgs vertex to a pair of charged leptons[6]. The electric charge is defined through the photon coupling to a positron-electron pair. We also set the residue of the photon and CP-odd Higgs equal to unity, and impose no mixing between the photon and the Z boson at zero momentum.

In Fig. 1 we plot the mixing angle $\alpha(p^2)$ as a function of the momentum p^2 for different values of the CP-odd Higgs mass m_A. For the top quark mass we take the CDF preferred value $m_t = 174\,\text{GeV}^{[7]}$. We take $\tan\beta = 25$ and all the squark soft supersymmetry breaking mass terms equal to 1 TeV. We consider no-mixing in the squark mass matrices ($\mu = A_U = A_D = 0$). The two crosses in each curve represent the masses of the two CP-even Higgs bosons. We see that the angle α at the scale m_h is very close to the one at the scale m_H, even for large splitting between these two masses, like in the case $m_A = 500 GeV$. In this later situation, the angle $\alpha \to 0$, consistent with the decoupling of the heavy Higgs (in general $\alpha \to \beta - \pi/2$, and in this case, $\beta \approx \pi/2$).

An important mechanism for the production of the neutral Higgs bosons in e^+e^- colliders is the brehmsstrahlung of a Higgs by a Z gauge boson. Relative to the coupling of the SM higgs to two Z bosons, the ZZh coupling is $\sin(\beta - \alpha)$. We plot this parameter in Fig. 2 as a function of $\tan\beta$. We contrast the tree level answer (dotted line) and the improved version (dashed line) defined by

$$\tan 2\alpha = \frac{(m_A^2 + m_Z^2)s_{2\beta}}{(m_A^2 - m_Z^2)c_{2\beta} + \Delta_t}, \quad \text{with} \quad \Delta_t = \frac{3g^2 m_t^4}{16\pi^2 m_W^2 s_\beta^2} \ln\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}, \quad (12)$$

with the parameter calculated with the momentum dependent mixing angle $\alpha(p^2)$. In this later case, we plot the result using $\alpha(p^2)$ evaluated at the two well motivated scales given by the masses of the two CP-even Higgs, and we find small differences between these two scales. However, important differences are found with the tree level and the improved cases, indicating that this effect may be important in the search of the Higgs boson at LEP.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

1) Momentum dependent mixing angle α (in degrees) as a function of $\sqrt{p^2}$, for the five different values of the CP-odd Higgs mass $m_A = 500, 130, 115, 80, 50$ GeV. We consider the CDF preferred value for the top quark mass and no squark mixing. All the other soft supersymmetry breaking mass terms are equal to 1 TeV. The two crosses over each curve correspond to the masses of the two CP-even Higgs bosons.

2) Coefficient $\sin(\beta - \alpha)$, the MSSM/SM ratio of the ZZh vertex, as a function of $\tan\beta$. We show the tree level value (dots), the improved value with the leading m_t^4 term (dotdash), and the value calculated with $\alpha(p^2)$ for two different choices of the squark mixing parameters and two different scales: $p^2 = m_H^2$ (dashes) and $p^2 = m_h^2$ (solid).